MATHEMATICAL MODELLING IN THE MALAYSIAN SECONDARY CURRICULUM

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Abstract

In this paper, the author discussed why mathematical modelling should be introduced in the mathematics curriculum at the secondary level in Malaysia. The importance of mathematical modelling process and the analysis on how Singapore and the USA had placed modelling in the mathematics curriculum are presented. Some modelling examples provided are meant to provide illustrations of the modelling tasks and processes that can be linked to the current Malaysian secondary curriculum. The benefits of modelling are enormous in the mathematics classroom. One not only understands how mathematics is used in the real-world but also construct a model or physical representation that is used to solve the problem. Using the modelling processes to solve real-world problems would help one in making conjectures and reasoning that is one of the important focus in the mathematics curriculum.

Introduction

In typical mathematics lessons the main focus is always to obtain the correct solution to a problem. Mathematical modeling, however, does not always have a simple and straight forward answer. Because of this, many educators have this question in mind on how do we teach the mathematical modeling process in tasks. Many articles had been written on the mathematical modeling process but the purpose of this article is to provide more insights on introducing mathematical modeling in the Malaysian curriculum (Ang, 2001; Blum & Leiss 2008; CCSSM 2010).

If mathematical modelling is so meaningful and useful then why is the implementation still lagging in the mathematics classroom? One possible reason is that mathematical modelling typically consists of challenging task that requires various competencies. There might be general steps in developing the model but no specific ways in solving them. They require real-world knowledge from domains that may not be as familiar to students and teachers, making their solutions less predictable and complex (Burkhardt, 2004; Ikeda, 2007). In addition modeling tasks are not like the typical "word problems" we have in textbooks that can be solved easily.

Mathematical modelling is the process of translating a real-life problem into a mathematical problem (Ang, 2001). Solving the modelling problem might not be simple as it usually involves integration of a few mathematical concepts. This may be one the challenges faced by teachers and students. Common Core State Standards in Mathematics (CCSSM) defines modelling as "the process of choosing and using appropriate mathematics and statistics to analyse empirical situations, to understand them better and to improve decisions" (CCSSM 2010, p. 72). Modelling deals primarily with solving the real-world problems without any

specific algorithm. This differs from mathematical problem solving which is a more guided task that can be solved (Ang, 2001). According to Blum (2011), mathematical modelling are tasks that requires the translation between reality and mathematics. A real modelling task would change a person's view on mathematics as a precise and accurate field to understanding the inaccuracies of measure methods and imprecise estimations in reality. Mathematics modelling can be summed up as "Mathematical modelling, on the other hand, begins in the "unedited" real world, requires problem formulating before problem solving, and once the problem is solved, moves back into the real world where the results are considered in their original context" (Pollak 2012, p. viii).

Malaysian Mathematics Curriculum

The Malaysian secondary mathematics curriculum aims to develop students with deep mathematical knowledge so that they can apply this knowledge in solving problems and making decisions (CDC, 2004). In the current Malaysian curriculum, the teaching and learning processes such as problem solving, communication, reasoning, making connections and the use of technology were emphasised. In fact, the development of problem solving skills is as an important area in the teaching and learning of mathematics in the curriculum. The four stages of problem solving heuristics by Polya (1945) that include understanding the problem, planning the strategy, carrying out the plan and reflecting on the solutions could be applied by students while solving problems in mathematics and daily lives. Integrating mathematical modelling into the existing mathematics curriculum seems possible and should be given consideration as it really exemplifies how mathematics is used in solving real-world problems.

Mathematical Modelling in Singapore and United States

Singapore students' mathematics achievement was among the top in the list as indicated in the international assessment studies for secondary mathematics such as TIMMS and PISA (Mullis 2012, OECD 2010). One of the factors contributing to this excellent result might be due to the Singapore mathematics curriculum. So it would be beneficial to examine the mathematics curriculum in Singapore and see whether modelling was included in the syllabus.

In Singapore, mathematical problem solving has been the central focus of the mathematics curriculum since the 1990s. The Singapore Mathematics Curriculum framework consists of five inter-related components such as the development of skills, concepts, attitudes, meta-cognition and processes. It is intended to produce students who are good problem solvers. One of the revisions made in the curriculum is the introduction of applications and modelling as one of the mathematical processes of the curriculum framework as well as mathematical reasoning, communication and making connections (Ministry of Education, 2007). The reason of this inclusion is "Through mathematical modelling, students learn to use a variety of representations of data, and to select and apply appropriate mathematical methods and tools in solving real-world problems" (Ministry of Education 2007, p.4). Besides problem solving, this provides Singapore teachers with an alternative instruction method to teach mathematics using real life problems.

Malaysian mathematics curriculum had been influenced strongly by the NTCM standards and processes. That is why it is worth looking at the development of the mathematics curriculum in the United States and whether mathematical modelling is given emphasis in the new CCSSM standards.

Problem solving and mathematical modelling are two related standards in the mathematics education arena. Mathematics educators around the world have been focusing on problem solving in the mathematics curriculum since the 1980s. In the United States, the National Council of Teachers of Mathematics (NCTM) produced the Evaluation Standards in 1989 and problem solving was one of the standards focused. The push for problem solving became even stronger with the document that asserted that "problem solving should be the central focus of the mathematics curriculum" (NCTM, 2000, p.52).

Mathematical modelling was highlighted in the document of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). Educational organisations such as Consortium for Mathematics and Its Application (COMAP), the University of Chicago School Mathematics Project (UCMSP) and the Systemic Initiative for Montana Mathematics and Science (SMMS) have produced many educational books and lessons on mathematics modelling (Hodgson, 1995). The development of the modelling standards for high schools mathematics in 2010 called the Common Core State Standards (CCSS) in the United State highlighted the importance of mathematical modelling.

Mathematics education in the United States is going through a change with the implementation of the Common Core State Standards (CCSSM) for mathematics in 2013. CCSSM is a state-led effort in collaboration with experts, school administrators and teachers with an aim to provide a clear and consistent framework to prepare the children for college and workforce .Forty-five states in the United States had adopted these standards in their mathematics curriculum. With standards, teachers are able to assist their students in acquiring the skills and knowledge they need to be successful. Modelling is one of the eight Standards included for Mathematical Practice in the CCSSM for all grades from K to 12 (CCSSM, 2010). Due to this, modelling could be the hands-on approach of teaching and learning mathematics. Modelling is also a required conceptual category in the high school curriculum that serves as a guide on how modelling can be used in enriching mathematical lessons. By using the integrated approach, mathematical modelling tasks require the use of other mathematical concepts in solving them. This makes it an efficient task during mathematical lessons.

Mathematical modelling process

To solve modelling problems, it is essential to construct a model of the situation. It might not be as simple as a regular problem solving tasks. Furthermore, real-world context problems do require adequate mathematical knowledge. With an explicit model of the modelling process, students are able to understand the process and develop strategies while attempting to solve the problem.

When the word model is mentioned, the first thing that comes to our mind is the image of a physical entity. Most of the time a model involves a scaled down version of an object or situation. The term mathematical model generally refers to the mathematical structure that is similar to a real-world problem or phenomenon. This active process of formulating a model is what we call as mathematical modelling (Swetz & Hartzler, 1991).

Figure 1 shows a simple mathematical modelling process. It is a simple process describing four modelling stages, namely, *Observation*, *Analysis*, *Interpretation and Application* (Ang, 1991; Swetz & Hartzler, 1991) although the terms used may differ according to researchers. All modelling process begins with the real world problem that can be formulated into

mathematical problems. The mathematical solutions obtained are usually interpreted in the real-world context before it can be accepted.

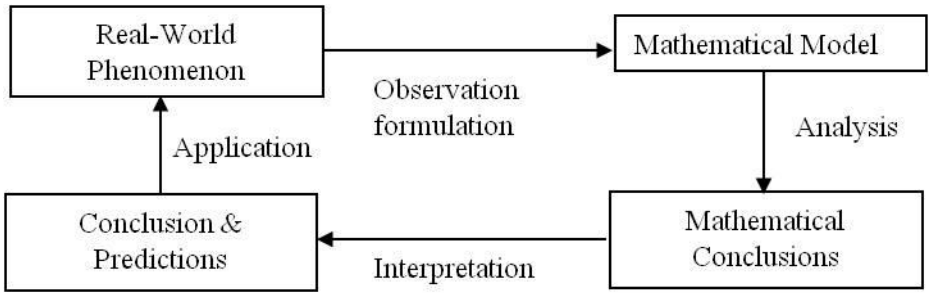

Figure 1. Mathematical Modelling Process.

In the CCSSM Standards 2010, a model can just be something simple to describe a product or relationship between variables. The modelling cycle begins with (CCSSM 2010, p.72)

> (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic or statistical representations that describe relationships between the variables, (3) analysing and performing operations of these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original solution, (5) validating the conclusion by comparing them with the situation, and then either improving or model or, if is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions and approximations are present throughout this cycle.

Figure 2 shows the mathematical modelling process described by the Common Core Standards in Mathematics (CCSSM, 2010).

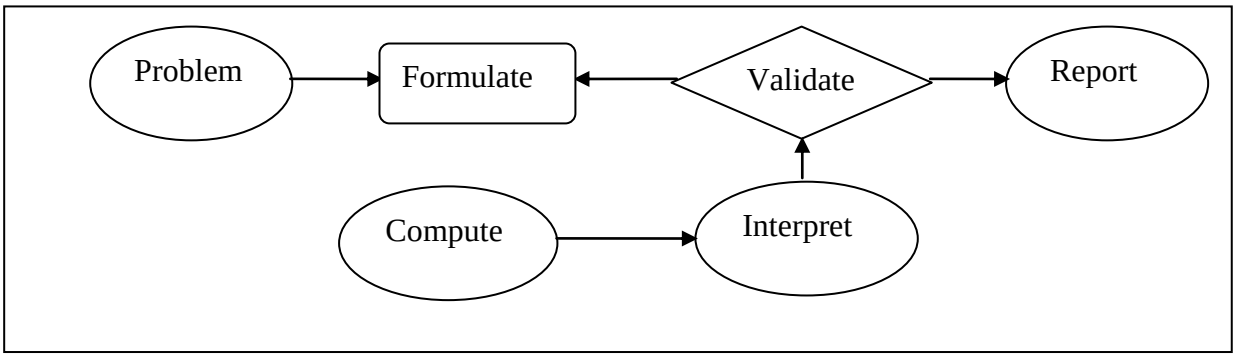

Figure 2. Mathematical modelling process by CCSSM (CCSSM 2010, p.72).

Examples of Modelling Task

This section provides one example of mathematical modelling task that can be used in the teaching and learning of mathematics. It would illustrate why the inclusion of the mathematical modelling in the Malaysian secondary curriculum should be implemented. This modelling task requires basic mathematical concepts and ideas across the field. It would be useful to introduce this type of task in the classroom. One way of implementing this task is by developing a modelling lesson based on this task. Students are given a worksheet with a leading question on the modelling task. Teachers could decide whether to implement the lesson using a combination of classroom discussion, group work and individual work. The

questions in the modelling task serves as a guide for students to develop their model with mathematics or sometimes with actual physical constructions. It can be a two day lesson with the first day focusing on creating the model and making assumptions. The second day lesson is the refining of the model to solve the task and ensuring that the model answers the question accurately (Gould, 2012). Sometimes the lesson could be extended beyond the original task or students could apply the model in a more complex situation. Figure 3 below is an example of the modelling task on birds and the guiding questions (Gould, 2012, p.2).

Task 1: For Birds

Your neighbour, an ornithologist, has to leave for the weekend to do a research study. She has asked you to make sure her birdfeeder always has food in it so that the birds keep coming back throughout the day. Refilling too seldom will cause the birds to look elsewhere for food; refilling too much will scare off the birds. How often should you feed the birds so they keep coming back?

Guiding Questions:

1. Your neighbour told you that it's important not to fill the feeder too often or to fill the feeder too seldom, so how can you determine how often to fill it?
2. When you go over first thing in the morning, the birdfeeder — which has 4 holes, one pair near the bottom and another pair about halfway up — is nearly full. You check back 45 minutes later and it's about half full. When do you expect it to empty again?

Figure 3. Modelling task and guiding questions.

This modelling task integrates three learning outcomes that include using units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays; define appropriate quantities for the purpose of descriptive modelling and rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (CCSSM, 2010). One needs to utilise previous mathematical knowledge to construct a physical representation of the model. Using different strategies and reasoning, a student would be able to relate this situation to rates and algebraic equation. Alternative ways of solving this real-life situation could be found by following the modelling cycle. The guiding question helps one to think of the assumptions and mathematics needed to find the solution (Gould, 2012).

Solving Modelling Task

The task below illustrates how the modelling task on the birdfeeder was solved. Not only does the task help one choose and interpret units consistently in formulas but it also enhances descriptive modelling. One would need to rearrange formulas to highlight the quantity of interest and use reasoning in solving equations to know how often the birds should be fed so that they will return. Figure 4 shows the modelling task.

Task 1: For Birds
Your neighbour, an ornithologist, has to leave for the weekend to do a research study. She has asked you to make sure her birdfeeder always has food in it so that the birds keep coming back throughout the day. Refilling too seldom will cause the birds to look elsewhere for food; refilling too much will scare off the birds. How often should you feed the birds so they keep coming back?

Figure 4. Modelling task.

Guiding Question:
When you go over first thing in the morning, the birdfeeder — which has 4 holes, one pair near the bottom and another pair about halfway up — is nearly full. You check back 45 minutes later and it's about half full. When do you expect it to empty again?

Figure 5. Modelling task guiding question.

Figure 5 shows the guiding questions on the modelling task that assist students in finding the solution to the task. To solve the birdfeeder problem, one has to go through the mathematical modelling process. Making assumptions is one of the key steps in solving the modeling task. It generally refers to identifying the important variables in the task. The variables that should be considered include the number of feeding holes in the birdfeeder, how fast does the portion of food empties and if the birds will always be feeding assuming that the bird are not afraid of human replenishing the food supply constantly. Now in the second guiding question, it is given is that the birdfeeder has 4 holes and it takes 45 minutes for the food level to be half full. The next step is the constructing of the mathematical model. This model can be a graph, equation, table, doing a simulation or a physical model. In this case, the mathematical model would be creating an equation with the defined variables.

To construct the equation, define

F = one feeder,
w = the rate at which the feeder empties (feeder/ minute)
t= time taken in minutes

The rate is always constant if $w = \frac{F}{t}$ and this gives $F = wt$. According to the information given, the rate changes at the middle of halfway point. Then we obtain the equation $F = w_1 t_1 + w_2 t_2$.

When the feeder is half-full after 45 minutes, the rate can be found by $\frac{1}{2}F = 45 \times w_1$. Solving the equation gives us the rate, $w_1 = \frac{1}{90}$. Next, we need to get the rate when the feeder is half-full till it is empty which is $w_2 = \frac{1}{2}w_1$. So the rate, $w_2 = \frac{1}{2}w_1 = \left(\frac{1}{2}\right)\left(\frac{1}{90}\right)$ is $w_2 = \frac{1}{180}$.

Then, we substitute all the values that we obtained into the equation $F = w_1 t_1 + w_2 t_2$. We get the value of $t_2 = 90$ minutes. So we would expect the feeder to be empty again in 135 minutes because $t_1 + t_2 = 45 + 90$. The mathematical model which is the equation in this case assisted us in solving the birdfeeder task.

Discussion

Mathematical modelling should be introduced in the Malaysian mathematics curriculum. Making modelling a curriculum standard increases the importance in the mathematics classroom similar to what is done in Singapore and in the United States of America. Adding modelling as one of the teaching and learning processes would develop the pedagogical strategies in a mathematics lesson.

With the introduction of the school-based assessment in the Malaysian secondary schools, mathematical modelling projects or tasks could be used to assess students' mathematical proficiency. This is because it applies mathematics ideas from different fields and students need to infer, make assumptions and test their models to solve the real-world problems. The learning of mathematics would be more realistic and it would really be a good example of how mathematics is used in our everyday's life.

Some challenges of implementing mathematical modelling in the curriculum would be endured. Issues such as training of teachers, time constraint, teachers' beliefs and students' engagement in the classroom would arise. Nonetheless, it provides an alternative instruction for mathematics teachers in their classroom. Furthermore, the modelling tasks could be used as a formative assessment in the school-based assessment currently being implemented in Malaysia. Modelling tasks would provide the hands-on experience of using mathematics in solving real–world problems similar to the experiments carried out in pure science classes. Training and exposure of mathematics modelling lessons would ensure that teachers understand when and how to implement the lessons. This enhances the teaching and learning processes in the Malaysian secondary curriculum that includes problem solving, communication, using technology, reasoning and critical thinking.

Teachers interested in the mathematics modelling would have to understand clearly what is the focus and meaning of this new idea and comparing it to problem solving. Without adequate support and resources such as modelling lessons and examples, teachers would find it difficult to teach mathematical modelling. They could look for resources which are available from the web since there are not much local materials available in Malaysia. Looking at resources from National Council of Teachers of Mathematics (NCTM) and topic study groups in the International Congress of Mathematics Education conference would be a good start. Teachers could also create and adapt mathematical lessons and tasks in the modelling context that are suitable for the students' mathematical ability.

While solving the modelling tasks, students can see the integration of different fields of mathematics such as algebra and geometry. This is different from the typical problem solving task that is placed at the end of chapter. One would expect students to use their existing mathematical knowledge to construct a model to solve the real-world problem.

The modelling tasks could be introduced to illustrate the mathematical knowledge needed. As the real-life situations, it might not be easy to solve them. Students might need some guidance in the beginning to formulate the problem before building the models. The assumptions applied should also be considered. Once this is done the process of solving and getting the solution is completed. Finally the solution is checked to see whether it makes sense and is applicable to the real-life scenario.

Conclusion

The benefits of modelling are enormous. One not only understands how mathematics is used in the real-world but also constructs a model or physical representation that is used to solve the problem. Using the modelling process to solve real-world problems would help one in making conjectures and reasoning which is one of the focuses of the mathematics curriculum. Mathematics lessons would be fun and challenging. This would develop further interest in this field. By going through the mathematical modelling tasks, one would develop higher order thinking in order to solve real world problems as well as applying problem solving skills in the process (Ang, 2009). This is why integrating mathematical modelling in the Malaysian secondary curriculum enhances the teaching and learning of mathematics in the classrooms. Even though there might be challenges in implementing mathematical modelling in the Malaysian curriculum, the benefits outweigh the problems. Sufficient planning, providing exposure and training of teachers would address the issue.

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